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A Simple Procedure for Estimating the Parameters of the Singh and Yadava Model for Fertility

Introduction

SINGH and Yadava (1977) have derived a model for number of conceptions *-Mo a female during a given period of time, assuming the start of the observational period at a considerable distance from marriage. The model is derived under the assumption that the period of non-susceptibility between any two consecutive births is constant and the female is fecund throughout the period of observation except during the non-susceptible period associated with a conception. The method of moment is used to estimate the parameters of the model,

In the present paper we have extended the above model by taking into consideration the variation in non-susceptibility period among females and assuming that some of them may be sterile also during the observational period. Further, we have given a method for estimation of parameters of the model which is more simple than the usual procedure discussed in earlier literature.

The usual procedure for estimation of parameters of such model is to equate the observed mean with the theoretical mean and the observed proportion of females with zero birth to the corresponding theoretical expression. Here we have obtained simple expressions for mean and probability of zero birth for the extended form of the model and used these expressions to estimate the parameters. Thus, our estimation procedure becomes more simple than the usual method used for the purpose. For illustration, the extended form of the model is applied to an observed set of data, taken from 'A Demographic Survey of

Varanasi (Rural) 1969-70' and the procedure is used to estimate the parameters of the model.

Model

Suppose that a female is married *at* time zero (0) and $X\ (T_0,\ T)$ dent›te the number of births to her during the period $(T_a,\ T_o\ +\ T)$, where T_0 is a distant point since marriage or menarche whichever is later. In the model of Singh and Yadava (1977) the nonsusceptibility period (h) is taken as constant:. We assume that this period, h, takes k values viz., h_1 $h_2, \ldots, h_k$ with probabilities respectively; and the probability that the female is fecund in the period of observation is so that 1 — a. is the probability that she is ste:rile.

It should be mentioned here that we have assumed one-to-one correspoi ıdence between a conception, and a live birth to avoid complexity in the model.

In the present case, the total number of conceptions, cannot be more tl ıan n, where $n = T/h_{min}$ or $[(T + h_{min})/h_{min}]$ accordingly as T is a multiple of h_{min} or not; where $[(T + h_{min})h_{min}]$ stands for the greatest integer not exceeding $(T +$ Amir, $)/h_{min}$ and h_{min} is the minimum of h_1 $(i = 1, 2, \ldots, k)$.

For fixed h, the probability distribution of $X\ (T_0\ T)$ is given as (see Singh and Yadava, 1977) :

$$P\{X(T_0, T) = 0\} = P_0 = \phi_1(0), \tag{1}$$

$$P\{X(T_0, T) = r\} = P_r = \phi_1(r) - 2\phi_1(r-1) + \phi_1(r-2), \quad r = 1, 2, \ldots, n-2 \tag{2}$$

$$P\{X(T_0, T) = n-1\} = P_{n-1} = n - \frac{mT}{1+mh} - 2\phi_1(n-2) + \phi(n-3), \tag{3}$$

$$P[X(T_0, T) = n] = P_n = -(n-1) + \frac{mT}{1+mh} + \phi_1(n-2), \tag{4}$$

where

$$\phi_1(r-1) = \frac{e^{-m(T-rh)}}{1+mh} \sum_{s=0}^{r-1} \sum_{k=0}^{s} \frac{\{m(T-rh)\}^k}{k!} .$$

Thus, taking variation in h and accounting for sterility, we have :

$$P[X(T_0, T) = 0] = P_0' = 1 - \alpha + \alpha \sum_{i=1}^{k} P[X(T_0, T) = 0/h = h_i]\ \pi_i; \tag{5}$$

and

$$P\,[X\,(T_0, T) = r] = P'_r = \alpha \sum_{i=1}^{k} P\,[X\,(T_0, T) = r/h = h_i]\, \pi_i, \quad r = 1, 2, \ldots, n \qquad (6)$$

Estimation

Singh and Yadava (1977) estimated m by equating the observed mean to theoretical mean and have obtained an explicit solution for m. Here the model involves parameters m, and the distribution of h. We have given a procedure to estimate the parameters m and assuming that the distribution of h is known. Two parameters viz. m and can be estimated by equating the observed mean with theoretical mean and the probability for the zero birth with the observed proportion of females with zero birth. We have

$$P'_0 = 1 - \alpha + \alpha \sum_{i=1}^{k} \pi_i \frac{e^{-m(T-h_i)}}{1 + mh_i} \;; \qquad (7)$$

and

$$E\,(X\,(T_0, T)) = \alpha \sum_{i=1}^{k} \pi_i \frac{mT}{1 + mh_i} \,. \qquad (8)$$

The above expressions seem to be a bit complicated. Using the results of Singh *et al.* (1979), we have obtained a simple expression for $E\,(X\,(T_0,\ T))$ as

$$E\,[X\,(T_0, T)] = \alpha \frac{mT}{1 + m\,\bar{h}'} \,, \qquad (9)$$

where

$$\bar{h}' = \sum_{i=1}^{k} \pi_i^* \; h_i, \qquad (10)$$

and

$$\pi_i^* = \frac{m\pi_i}{1 + mh_i} \Big/ \sum_{i=1}^{k} \frac{m\pi_i}{1 + mh_i} \,. \qquad (11)$$

Some test calculations have shown that the expression

$$\sum_{i=1}^{k} \pi_i \frac{e^{-m(T-h_i)}}{1 + m\,h_i},$$

can be approximated by

$$\frac{e^{-m(T-\bar{h}')}}{1 + m\bar{h}'} .$$

Equating the observed proportion of couples with zero birth with its theoretical expression (7) and utilizing the above approximation, we have

$$\frac{N_0}{N} = 1 - \alpha + \alpha \frac{e^{-m(T-\bar{h}')}}{1 + m\,\bar{h}'} , \tag{12}$$

where N_0 and N represent the number of females having no birth during (T_0, T) and total number of females under consideration respectively.

Combining equations (9) and (12), we get a *simple* equation which involves only m and is independent of α. The equation is

$$T\left(\frac{N - N_0}{N\bar{X}}\right) = \frac{1}{m} + \bar{h}' - \frac{e^{-m(T-\bar{h}')}}{m}, \tag{13}$$

where $\bar{X}$ *is the observed mean* of $X(T_0, T)$. The parameter m can easily be estimated from (13) provided $\bar{h}'$ is known. Singh *et al.* (1979) have shown that $\bar{h}'$ is not much affected by small changes in m. Hence, a trial value of m can be obtained by solving

$$T\left(\frac{N - N_0}{N\bar{X}}\right) = \frac{1}{m} + \bar{h} - \frac{e^{-m(T-\bar{h})}}{m} \tag{14}$$

where

$$\bar{h} = \sum_{i=1}^{k} \pi_i\, h_i.$$

From this trial value of m we compute $\bar{h}'$ from (10) and (11), and substituting this value of $\bar{h}'$ in (13) we get a more refined estimate of m. The estimate of α *is obtained from* (12) *with the knowledge of the value of* m.

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From this trial value of *m* we compute $\bar{h}'$ from (10) and (11), and substituting this value of $\bar{h}'$ in (13) we get a more refined estimate of *m*. The estimate of a is obtained from (12) with the knowledge of the value of *m*.

Application

For illustration, the model has been applied to an observed distribution taken from 'A Demographic Survey of Varanasi (Rural) 1969-70' For detailed account of the survey see Singh *et al,* (1970).

Table 1 presents the observed distribution of number of births in last 7 years to females aged 25-29 years. In the surveyed area, the average age at marriage is nearly 15 years and the observed distribution relates to females aged 25-29 years. **Hence** the start of the observational period *may* be assumed at a considerable distance from marriage.

TABLE 1—DISTRIBUTION OF FEMALES AGED 25-29 YEARS **ACCORDING TO** THE NUMBER OF BIRTHS DURING THE LAST SEVEN **YEARS**

Number of births	*Observed number of women*	*Expected number of women*
0	**23**	**23.0**
1	**57**	**59.3**
2	**161**	**167.2**
3	**148**	**132.1**
4	*32*	**36.0**
5	**3**	**6.4**
Total	**424**	**424.0**

X^2 = 4.48356 (d.f. = 3).

The non-susceptible period consists of gestation plus post-partum amenorrhea (p.p.a.). The analysis of the pattern of p.p.a., by Singh and Bhaduri (1971) has revealed that the nature of the curve is bimodal, the first and second peaks occurring at 2-3 months and 12-13 months respectively. They have also observed that the proportion of females belonging to smaller and larger p.p.a, groups are approximately 0.4 and 0.6 respectively. Keeping in view these findings, we take = 0.4, H_1 — 1 year and h_2 — 1.75 years.

A trial value of *m* is obtained by solving (14) which comes out 0.6705. Using equations (10) and (11) the values of n^* and $\bar{h}$ are obtainid as 0.4645 and 1.401625 respectively. Now substituting this value of $\bar{h}$ in equation (13) we get a more refined estimate of *m,* as $\hat{m}$ - 0.6471. Further, using equation (5) and equating this with the observed proportion of females with zero birth during (T_o, T), we get $\hat{}$ = 0.95959.

The expected frequencies arc shown in column (3) of the table. The value of X^2 is insignificant at 5% level of significance. The proposed model seems to be a reasonable approximation for the observed situation.

References

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